Comments on Babinet's Principle

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Abstract—**A one-dimension transmission line model is employed to emulate the scattering behavior of electromagnetic waves incident upon complementary screens. It is found that, when Babinet's principle conditions are met, both equivalent electric terminal impedances of complementary screens are equal and negative in real values and their nature is not known. It is, therefore, pertinent to re-examine the Babinet's Principle, pinpoint its features, as well as its limitations.**

Index Terms—**Babinet's principle, Transmission line theory, Complementary structure.**

I. INTRODUCTION

Electromagnetic/light waves send information to and from structure of materials, and provide a way for non-invasive detection of physical structures of media. It can also package data in a signal of zero mass with unmatched speed. The underlying principles of propagation and scattering of electromagnetic/light wave have been discussed by many scientists since ancient times [1]-[2]. More recently, some researchers [3]-[28] have employed Babinet's principle to interpret electromagnetic/optical wave properties in a variety of media. Babinet's principle deals with the distribution of waves diffracted by an obstructed body and its complementary structure. Babinet's principle has largely been employed to compare the diffraction pattern of an aperture with that of a complementary disk. Shalaev [3] reviewed optical negative-index meta-materials, wherein, based on Babinet's principle, similar resonance behavior for negative index material could be obtained for voids and nanorod arrays. Li [4] reported strong field enhancement around the center of the uniform mesoporous gold film. This phenomenon can be explained by applying the Babinet's principle to the reverse system of nanoparticle ensemble. Jong [5] proposed an anisotropic model for asymmetrical meta-atom clusters and their polarization dependency, in which the complementary configurations of meta-atom structures showed the same behavior of the resonance frequencies as predicted by the Babinet's principle. In 2004, Falcone [6] used the electromagnetic theory of diffraction and the Babinet's principle to design artificial meta-surfaces and meta-materials that have negative dielectric permittivities. In radio wave engineering, Booker [7] related the radiated fields as well as the impedance of an aperture, or a slot antenna, to those of its complementary dipole antenna realized by interchanging the

the conductive material and air regions of the slot antenna. Booker obtained the relationship between the terminal impedances of a slot antenna and its complementary dipole antenna. In this communication, we investigate the wave scattering characteristics of an obstructed body and its complement by using a transmission-line model and terminal impedances of the obstructed body and its complement, focusing on issues relevant to the Babinet's principle.

II. THEORY

Babinet's principle is a theorem in physics pertaining to the diffraction of electromagnetic waves. The principle, which was originally introduced in optics [1], was later extended to electromagnetic radiation as described below. Let D be the original diffracting body, and let D' be its complement, *i.e.*, a body which is transparent in regions where D is opaque, and *vice versa*. The sum of the scattering patterns of D and D' must be identical to the scattering pattern of the unobstructed beam. In the regions where the undisturbed beam would not have been reached, the scattering patterns of D and D' must be opposite in phase, but equal in amplitude in those regions.

To illustrate the principle, we show in Fig. 1 [29] a representative configuration which consists of a source, an obstructed body or its complement, and the observation screen. Fig. 1(a) shows a source, a screen D as the obstructed body (diffracting body), and an observation screen S; Fig. 1(b) shows a source, a complementary screen D', and an observation screen S; Fig. 1(c) shows a source, a virtual screen (no screen), and the observation screen S; Fig. 1(d) [29]-[30] shows the transmission-line representation of the above three systems, where Z_0 is the equivalent wave impedance of free space, Z_1 is the equivalent shunt load impedance of obstructed screen in Fig. 1(a), Z_2 is the equivalent shunt load impedance of complementary screen in Fig. 1(b). The sum of transmission patterns caused by the obstructed body and its complement must be the same as the transmission patter appearing on the observation screen when no obstructed screen is present, as shown in Fig. 1(c). Hence, we can write :

$$Z_1 Z_2 = \frac{Z_0^{\,2}}{4}. \qquad (1)$$

Equation (1) can be found in books on antennas, as well as in journal articles [7], [29], [31]. The above equation is most widely used to relate the impedances of complementary structures, *e.g.*, dipole and loop antennas. The equation can be obtained from the transmission properties of electromagnetic waves, namely that the sum of transmission patterns of the obstructed screen and its complement is the same as that of the unobstructed wave; hence, (1) is an implicit relationship that can be used to find the equivalent impedances of the obstructed screen and its complement. On the reflection side where the undisturbed wave would not have been reached, the reflected patterns caused by **D** and **D'** must be opposite in phase, but equal in amplitude. This leads us to the condition:

$$S_{11,1} = - S_{11,2,}\tag{2}$$

where $S_{11,1}$ and $S_{11,2}$ are the reflection coefficients of the transmission-line representations of the structures shown in Figs.1(a) and 1(b), respectively, and they can be written [32]-[34]

$$S_{11,i} = \frac{-Z_0}{2Z_i + Z_0},\tag{3}$$

with i = 1, 2. Upon substituting (3) into (2), we obtain

$$Z_1 + Z_2 + Z_0 = 0.\tag{4}$$

Equation (4) is the relationship between the wave impedance in space and the shunt equivalent impedances of the obstructed and complementary bodies, if the reflected fields caused by the obstructed body and its complement are opposite in phase, but equal in amplitude, as they must be. It is pertinent to point out that the wave impedance of free space is generally real and positive. Therefore, the sum of real parts of Z_1 and Z_2 is negative. Upon substituting (4) into (1), we obtain

$$Z_1 = Z_2 = -\frac{Z_0}{2},\tag{5}$$

for the lossless case.

Note that (5) is valid when the Babinet's principle is applied to the lossless structure by using the transmission line model under the conditions of both the total transmission wave, and the zero-sum reflection caused by the obstructed and complementary bodies. Equation (5) also reveals that the structures of obstructed body and complementary body could be self-complementary. We do not know what the obstructed and the complementary bodies look like, but only that the equivalent shunt impedances of the obstructed body and its complement are equal.
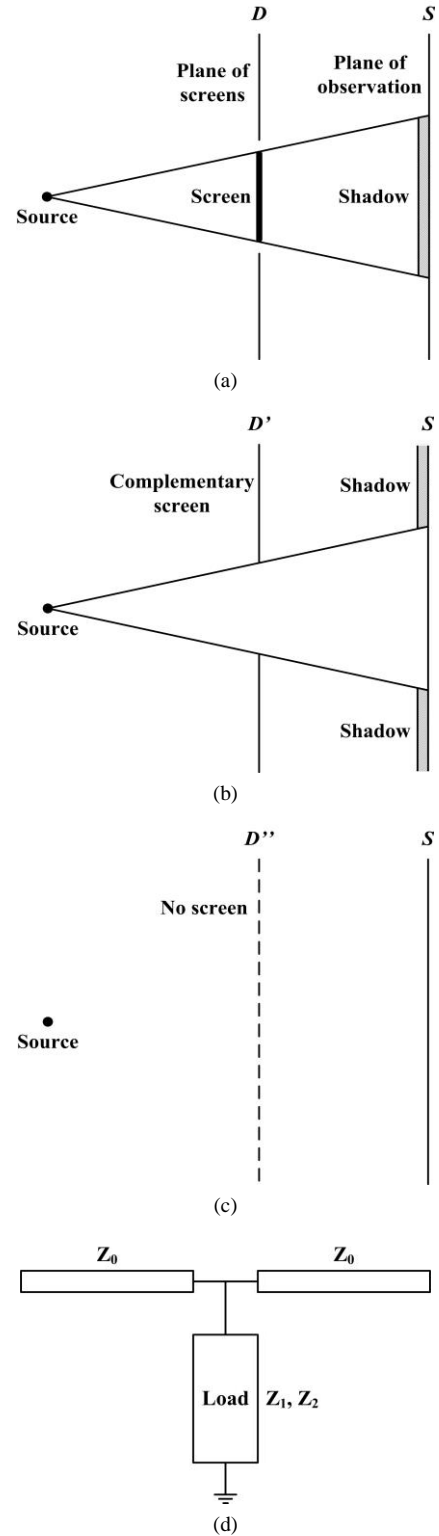

Fig. 1. Illustration of Babinet's principle. (a) A perfect absorbing screen in plane *D*; (b) A complementary screen in plane *D'*; (c) No screen; (d) Transmission-line equivalent circuit of the above three systems.

III. Conclusions

We will now conclude with the following observations: If the sum of the scattered fields caused by an obstructed body and its complement is the same as the scattered field of the unobstructed beam, and if in places where the undisturbed

beam would not have been reached the scattered fields caused by an obstructed body and its complement are opposite in phase, but equal in amplitude – in accordance with the Babinet's principle - then the electric terminal impedances of both the obstructed body and its complement are negative resistances equal to $-Z_0/2$, where Z_0 is the wave impedance of the transmission medium (or free space). While the structures of the obstructed body and its complement are self-complementary, the nature of such structures needs not be known.

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